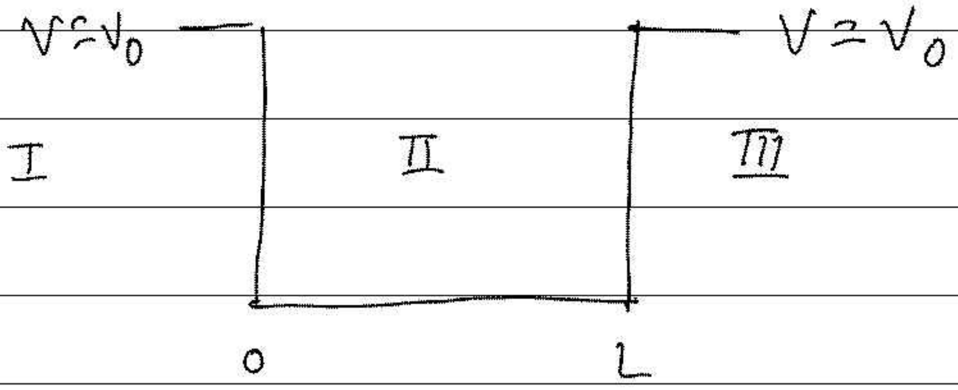
Finite Potential Well



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V_0 - E)\psi \quad \ldots (1)$$

$$k_1^2 = \frac{2m}{\hbar^2}(V_0 - E) \quad \ldots (2)$$

Region I : $\psi_I = Ae^{-k_1x} + Be^{k_1x}$ ... (3)

Region III : $\psi_{III} = Fe^{-k_1x} + Ge^{k_1x}$ ... (4)

Region II : $\psi_{II} = C\sin k_2x + D\cos k_2x$ ... (5)

Region I:

$$\psi_{I} = Ae^{-k_1 x} + Be^{k_1 x} \qquad x<0$$

$$\psi_{I} = Be^{k_1 x} \quad \ldots \quad (6)$$

Region III

$$\psi_{III} = Fe^{-k_1 x} + Ge^{k_1 x} \qquad x>L$$

$$\psi_{III} = Fe^{-k_1 x} \quad \ldots \quad (7)$$

Continuity:

$$\left.\begin{aligned} \psi_{I}(x=0) &= \psi_{II}(x=0) \\ \psi_{II}(x=L) &= \psi_{III}(x=L) \end{aligned}\right\} (8)$$

$$Be^{k_1 x} = C\sin k_2 x + D\cos k_2 x$$

$x=0 \qquad B = D \quad \ldots \quad (9)$

Continuity of slope:

$$\frac{d\psi_I}{dx}(x=0) = \frac{d\psi_{II}}{dx}(x=0)$$

$$k_1 B e^{k_1 x} = k_2 C \cos k_2 x - k_2 D \sin k_2 x$$

$x=0$

$$k_1 B = k_2 C \quad \cdots \quad (10)$$

Normalitation:

$$\int_{-\infty}^{0} \psi_I^2 \, dx + \int_{0}^{L} \psi_{II}^2 \, dx + \int_{L}^{\infty} \psi_{III}^2 \, dx = 1$$

$$k_1^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\left.\begin{aligned} \psi_{III} &= F e^{-k_1 x} \\ k_1 &= \sqrt{\frac{2m}{\hbar^2}(V_0 - E)} \end{aligned}\right\} \rightarrow \text{Imp!!}$$